A NOTE ON REGULARITY FOR DISCRETE ALLOY-TYPE MODELS

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Abstract. Standard proofs of localization for random Schrödinger operators require certain regularity conditions on the random potential. In this informal note we discuss wheather the discrete alloy-type potential is uniformly τ-Hölder continuous or not.

1. Introduction and model

In [And58] P. W. Anderson introduced a simple model that describes a spinless electron which moves in a static random electric potential; the so called *Anderson model*. Starting with this model random operators became increasingly an interesting area in the numerical and analytical research. The Anderson model is the random discrete Schrödinger operator $H = -\Delta + \lambda V$ acting in $\ell^2(\mathbb{Z}^d)$. Here Δ denotes the discrete Laplacian, V is a random potential given by a collection of random variables $\{V(x)\}_{x\in\mathbb{Z}^d}$, and $\lambda > 0$ controles the strength of the *disorder*. In the "original" Anderson model the random potentials are assumed to be independent identically distributed (i. i. d.) random variables, each distributed uniformly on the interval $[-1/2, 1/2]$. Anderson proposed in [And58] that the randomness of the potential leads to localization phemonena in the solutions of the Schrödinger equation. This localization phenomena manifests for example in the fact that the operator H obeys almost surely only pure point spectrum in the case of sufficiently large disorder. This phenomena was first proven by Fröhlich and Spencer in [FS83] where they laid the foundation of *multiscale analysis*. This method then was further developed, e. g., in [FMSS85, vDK89]. Later, Aizenman and Molchanov introduced in [AM93] a different method to prove localization; the *fractional moment method*. These proofs of localization also apply to more general random potentials by imposing certain regularity conditions on the distribution of the random potential. Also the case where the potential values at different lattice sites are correlated random variables was treated [AM93, vDK91, AG98, Hun00, ASFH01, Hun08].

Let us now introduce class of potentials we are interested in. Let the *single-site potential* $u : \mathbb{Z}^d \to \mathbb{R}$ be a function with compact support $\Theta := \operatorname{supp} u = \{k \in \mathbb{Z}^d : u(k) \neq 0\}$ and $0 \in \Theta$. Furthermore, let us introduce the probability space $\Omega := \times_{k\in\mathbb{Z}^d} \mathbb{R}$ equipped with the probability measure $\mathrm{d}\mathbb{P}(\omega) := \prod_{k\in\mathbb{Z}^d} \rho(\omega_k)\mathrm{d}\omega_k$ where $\rho \in L^\infty(\mathbb{R}) \cap L^1(\mathbb{R})$ with $\|\rho\|_{L^1} = 1$. Hence, each element ω of Ω may be represented as a collection $\{\omega_k\}_{k\in\mathbb{Z}^d}$ of i. i. d. random variables, each distributed with the density ρ. We introduce the *discrete alloy-type potential* $V_\omega : \mathbb{Z}^d \to \mathbb{R}$ by

$$V_\omega(x) = \sum_{k\in\mathbb{Z}^d} \omega_k u(x - k). \tag{1}$$

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The Schrödinger operator $H_\omega : \ell^2(\mathbb{Z}^d) \to \ell^2(\mathbb{Z}^d)$, defined by

$$H_\omega = -\Delta + \lambda V_\omega, \quad \lambda > 0,$$

is called *discrete alloy-type model*. Here, Δ denotes the discrete Laplacian given by $(\Delta\psi)(x) = \sum_{|e|=1} \psi(x+e)$ and V_ω is the multiplication operator by the function in Eq. (1). Operators of this form have been studied for example in [Ves10, ETV10, Krü10].

2. A regularity condition for correlated random fields

Anderson models where the potential values at different lattice sites are not independent have been studied previously in the literature according to the multiscale analysis and the fractional moment method, see e. g. [AM93, vDK91, AG98, Hun00, ASFH01, Hun08]. Among others, they proof localization as long as the potenial values satisfy certain regularity conditions. More presicely, they require regularity of the distribution of the potential at $x \in \mathbb{Z}^d$ conditioned on arbitrary fixed potential values elsewhere. One question is, wheather the regularity conditions of the above mentioned papers are satisfied for the discrete alloy-type potential or not, in other words, wheather the theorems in [AM93, vDK91, AG98, Hun00, ASFH01, Hun08] apply to our model or not. To be specific, let us formulate the regularity condition from [ASFH01].

Definition 2.1. Let X be a countable set, $\{v_x\}_{x\in X}$, be a collection of random variables and $\rho_x(\cdot \mid v_x^\perp)$ the probability distribution of v_x conditioned on the random variables $v_x^\perp = \{v_j\}_{j\in X\setminus\{x\}}$. The collection $\rho_x(\cdot \mid v_x^\perp)$, $x \in X$, is said to be (uniformly) *τ-Hölder continuous* for $\tau \in (0,1]$ if there is a constant C such that

$$\sup_{x\in X} \sup_{v_x^\perp} \rho_x([a,b] \mid v_x^\perp) \leq C(b-a)^\tau \quad \text{for all } [a,b] \subset \mathbb{R}.$$

The second supremum is taken over all possible values of $v_x^\perp$ in $\times_{\mathbb{Z}^d\setminus\{x\}}\mathbb{R}$.

3. The case $\operatorname{supp}\rho$ compact; a counter-example

In the following we want to study in which cases the regularity condition of Definition 2.1 is satisfyed for the alloy-type potential and for which not. First we give a result in the negative direction.

Lemma 3.1. *Let $d = 1$, $\Theta = \{0, 1, \dots, n-1\}$ for some $n \in \mathbb{N}$,* $\inf\operatorname{supp}\rho = 0$ *and* $\sup\operatorname{supp}\rho = 1$. *Then there are constants $c, m, s^+ \in (-\infty, \infty)$, depending only on u, such that for all $\delta > 0$ and $\delta \geq \delta' > 0$*

$$\mathbb{P}\bigl\{V_\omega(0) \in [m - c\delta, m + c\delta] \mid V_\omega(-1), V_\omega(n-1) \in [s^+ - \delta', s^+]\bigr\} = 1.$$

The values of the constants c, m and s^+ can be inferred from the proof.

Notice that, under the assumptions of Lemma 3.1, $V_\omega(-1)$ and $V_\omega(n-1)$ are stochastically independent and $\mathbb{P}\{V_\omega(-1), V_\omega(n-1) \in [s^+ - \delta', s^+]\} > 0$, where s^+ is defined in the proof of Lemma 3.1.

Proof of Lemma 3.1. Let $\Theta^+ := \{k \in \mathbb{Z} : u(k) > 0\}$ and $\Theta^- := \{k \in \mathbb{Z}^1 : u(k) < 0\}$. Further let $u_{\max} = \max_{k\in\Theta}|u(k)|$, $u_{\min} = \min_{k\in\Theta}|u(k)|$ and $s^+ = \sum_{k\in\Theta^+} u(k)$.

Let us introduce two further subsets of Θ which are important in our study. The first one is

$$\Theta_1 = \begin{cases} \Theta^+ + 1 & \text{if } n-1 \notin \Theta^+, \\ ((\Theta^+ + 1) \cap \Theta) \cup \{0\} & \text{if } n-1 \in \Theta^+, \end{cases}$$

with $\Theta^+ + 1 = \{k \in \mathbb{N} : (k-1) \in \Theta^+\}$. The second subset is the complement $\Theta_0 = \Theta \setminus \Theta_1$. To end the proof we show the following interval arithmetic result:

Let $\delta \geq \delta' > 0$ and $V_\omega(-1), V_\omega(n-1) \in [s^+ - \delta', s^+]$. Then

$$V_\omega(0) \in [m - c\delta', m + c\delta'] \subset [m - c\delta, m + c\delta] \tag{2}$$

with $c = n u_{\max}/u_{\min}$ and $m = \sum_{k \in \Theta_1} u(k)$.

We devide the proof of (2) into three parts. The first step is to argue that

$$\omega_{-1-k} \in \begin{cases} \left[1 - \frac{\delta'}{u_{\min}}, 1\right] & \text{for } k \in \Theta^+, \\ \left[0, \frac{\delta'}{u_{\min}}\right] & \text{for } k \in \Theta^-. \end{cases} \tag{3}$$

For the proof of the first part of (3) we use the assumption $V_\omega(-1) \geq s^+ - \delta'$ and obtain

$$s^+ - \delta' \leq V_\omega(-1) = \sum_{k \in \Theta} u(k)\omega_{-1-k} \leq \sum_{k \in \Theta^+} u(k)\omega_{-1-k},$$

and hence $\sum_{k \in \Theta^+} u(k)(1 - \omega_{-1-k}) \leq \delta'$. We conclude that for all $k \in \Theta^+$ we have $u(k)(1 - \omega_{-1-k}) \leq \delta'$ which gives the first part of (3). For the proof of the second part of (3) we use again the assumption $V_\omega(-1) \geq s^+ - \delta'$ and obtain

$$\sum_{k \in \Theta^+} u(k)\omega_{-1-k} - \delta' \leq s^+ - \delta' \leq V_\omega(-1) = \sum_{k \in \Theta^+} u(k)\omega_{-1-k} + \sum_{k \in \Theta^-} u(k)\omega_{-1-k}$$

which gives $-\delta' \leq \sum_{k \in \Theta^-} u(k)\omega_{-1-k}$. Thus, for all $k \in \Theta^-$ we have $\omega_{-1-k} \leq -\delta'/u(k) = \delta'/|u(k)|$ which gives the second part of (3). In a second step we argue that

$$\omega_{-k+n-1} \in \begin{cases} \left[1 - \frac{\delta'}{u_{\min}}, 1\right] & \text{for } k \in \Theta^+, \\ \left[0, \frac{\delta'}{u_{\min}}\right] & \text{for } k \in \Theta^-. \end{cases} \tag{4}$$

The proof of (4) can be done in analogy to the proof of (3), but using the assumoption $V_\omega(n-1) \geq s^+ - \delta'$. In a third step we ask the question for which $k \in \Theta$ we have $\omega_{-k} \in [1 - \delta'/u_{\min}, 1]$. Using the definition of the set Θ_1 we find with (3) and (4) that

$$\omega_{-k} \in \begin{cases} \left[1 - \frac{\delta'}{u_{\min}}, 1\right] & \text{for } k \in \Theta_1, \\ \left[0, \frac{\delta'}{u_{\min}}\right] & \text{for } k \in \Theta_0. \end{cases} \tag{5}$$

Now, the desired result (2) follows from (5) and the decomposition

$$V_\omega(0) = \sum_{k \in \Theta} u(k)\omega_{-k} = \sum_{k \in \Theta_1} u(k)\omega_{-k} + \sum_{k \in \Theta_0} u(k)\omega_{-k}.$$

Hence, the proof is complete. ■

Remark 3.2. The assumption $\inf \operatorname{supp} \rho = 1$ and $\sup \operatorname{supp} \rho = 1$ in Lemma 3.1 is not crucial. What matters is that $\operatorname{supp} \rho$ is a bounded set.

Remark 3.3. Lemma 3.1 implies that the collection of random variables $V_\omega(k)$, $k \in \mathbb{Z}^d$, is not uniformly τ-Hölder continuous. Hence, the results in [ASFH01] do not apply to the discrete alloy-type model in general.

4. The Gaussian case

Now, we consider the case $d = 1$, $\Theta = \{-1, 0\}$, $u(0) = 1$ and where ρ is a Gaussian density function with mean zero and variance σ^2. In this situation it turns out that the regularity assumption from [ASFH01] is satisfied as long as $|u(-1)| \neq 1$. Our study is based on following classical result which may be found in [Por94].

Proposition 4.1. *Let X be normally distributed on $\mathbb{R}^d$, $Y = a \cdot X$ where $a \in \mathbb{R}^d$, and $W = BX$ where $B \in \mathbb{R}^{m \times d}$. Assume W has a nonsingular distribution. Then the distribution of Y conditioned on $W = v \in \mathbb{R}^m$ is the Gaussian distribution having mean*

$$\mathbf{E}(Y) + \operatorname{cov}(Y, W) \operatorname{cov}(W, W)^{-1} [v - \mathbf{E}(W)]$$

and variance

$$\operatorname{cov}(Y, Y) - \operatorname{cov}(Y, W) \operatorname{cov}(W, W)^{-1} \operatorname{cov}(W, Y).$$

For $l \in \mathbb{N}$ let $A_l \in \mathbb{R}^{l \times l+1}$ be the matrix with coefficients in the canocical basis given by $A_l(i, i) = 1$, $A_l(i, i+1) = u(-1)$ for $i \in \{1, \dots, l\}$, and zero otherwise, namely

$$A_l = \begin{pmatrix} 1 & u(-1) & & & \\ & \ddots & \ddots & & \\ & & \ddots & u(-1) & \\ & & & 1 & u(-1) \end{pmatrix} \in \mathbb{R}^{l \times l+1}.$$

Notice, if we apply A_l on the vector $\omega_{[x,x+l]} = (\omega_{x+k-1})_{k=1}^{l+1}$, we obtain a vector containing the potential values $V_\omega(k)$, $k \in \{x, x+1, \dots, x+l\}$. Moreover, the vector $(V_\omega(x+k-1))_{k=1}^{l} = A_l \omega_{[x,x+l]}$ is normally distributed with mean zero and covaniance $\sigma^2 A_l A_l^{\mathrm{T}}$. The matrix $A_l A_l^T$ has the form

$$A_l A_l^T = \begin{pmatrix} 1 + u^2(-1) & u(-1) & & \\ u(-1) & 1 + u^2(-1) & \ddots & \\ & \ddots & \ddots & u(-1) \\ & & u(-1) & 1 + u^2(-1) \end{pmatrix} \in \mathbb{R}^{l \times l}.$$

By induction we find that the determinant of $A_l A_l^{\mathrm{T}}$ is given by

$$\det(A_l A_l^{\mathrm{T}}) = s_l > 0 \quad \text{where} \quad s_l := \sum_{i=1}^{l} \bigl(u(-1)\bigr)^{2i}.$$

Since the minor M_{11} and M_{ll} of $A_l A_l^{\mathrm{T}}$ equals $A_{l-1} A_{l-1}^{\mathrm{T}}$ we obtain by Cramers rule for the elements $(1, 1)$ and (l, l) of the inverse of $A_{l-1} A_{l-1}^{\mathrm{T}}$

$$(A_l A_l^{\mathrm{T}})^{-1}(1, 1) = (A_l A_l^{\mathrm{T}})^{-1}(l, l) = \frac{s_{l-1}}{s_l}. \tag{6}$$

Lemma 4.2. *Let $d = 1$, $l, m \geq 1$, $\Theta = \{-1, 0\}$, $u(0) = 1$ and ρ be the Gaussian density with mean zero and variance σ^2. Let further $v^+ \in \mathbb{R}^l$ and $v^- \in \mathbb{R}^m$. Then the distribution of $V_\omega(0)$ conditioned on $(V_\omega(k))_{k=1}^l = v^+$ and $(V_\omega(-m+k-1))_{k=1}^m = v^-$ is Gaussian with variance*

$$\gamma = \sigma^2\Big(u(-1)^2 - 1 + \frac{1}{s_m} + \frac{1}{s_l}\Big)$$

and mean

$$m = u(-1)\left(\sum_{i=1}^{m}(A_m A_m^{\mathrm{T}})^{-1}(m,i)\, v_i^- + \sum_{i=1}^{l}(A_l A_l^{\mathrm{T}})^{-1}(1,i)\, v_i^+\right).$$

Proof. Let $X := (\omega_{-m-1+k})_{k=1}^{l+m+2} \in \mathbb{R}^{m+n+2}$, $a = (a_i)_{i=1}^{l+m+2} \in \mathbb{R}^{l+m+2}$ the vector with coefficients $a_{m+1} = 1$, $a_{m+2} = u(-1)$ and zero otherwise. Let us further define the block-matrix

$$B = \begin{pmatrix} A_m & 0 \\ 0 & A_l \end{pmatrix} \in \mathbb{R}^{(m+l)\times(m+l+2)}.$$

Notice that $Y := a \cdot X = V_\omega(0)$,

$$A_m \omega_{[-m,0]} = (V_\omega(-m+k-1))_{k=1}^m, \quad \text{and} \quad A_l \omega_{[1,l+1]} = (V_\omega(k))_{k=1}^l,$$

where $\omega_{[-m,0]} = (\omega_{-m+k-1})_{k=1}^{m+1}$ and $\omega_{[1,l+1]} = (\omega_k)_{k=1}^{l+1}$. Hence $W := BX$ is the $m+l$-dimensional vector containing the potentials $V_\omega(k)$, $k \in \{-m, \dots, l\} \setminus \{0\}$. Notice that Y and W have mean zero, since X has mean zero. We apply Proposition 4.1 with these choices of X, Y and W, and obtain that the distribution of $V_\omega(0)$ conditioned on $(V_\omega(-m+k-1))_{k=1}^m = v^-$ and $(V_\omega(k))_{k=1}^l = v^+$ is Gaussian with mean

$$m = \operatorname{cov}(Y, W)\operatorname{cov}(W, W)^{-1} v$$

and variance

$$\gamma = \operatorname{cov}(Y, Y) - \operatorname{cov}(Y, W)\operatorname{cov}(w, w)^{-1}\operatorname{cov}(W, Y),$$

where $v = (v^-, v^+)^{\mathrm{T}}$. It is straightforward to calculate $\operatorname{cov}(Y, Y) = \sigma^2(1 + u(-1)^2)$ and $\operatorname{cov}(W, Y) = z = (z^-, z^+)^{\mathrm{T}}$, where $z^- = (0, \dots, 0, \sigma^2 u(-1))^{\mathrm{T}} \in \mathbb{R}^m$ and $z^+ = (\sigma^2 u(-1), 0, \dots, 0)^{\mathrm{T}} \in \mathbb{R}^l$. We also have

$$\operatorname{cov}(W, W) = \sigma^2 \begin{pmatrix} A_m A_m^{\mathrm{T}} & 0 \\ 0 & A_l A_l^{\mathrm{T}} \end{pmatrix}.$$

Hence by Eq. (6)

$$\begin{aligned}
\gamma &= \sigma^2(1 + u(-1)^2) - \sigma^{-2} z^{\mathrm{T}} \begin{pmatrix} (A_m A_m^{\mathrm{T}})^{-1} & 0 \\ 0 & (A_l A_l^{\mathrm{T}})^{-1} \end{pmatrix}^{-1} z \\
&= \sigma^2(1 + u(-1)^2) - \sigma^{-2}\left[\sigma^4 u^2(-1)\frac{s_{m-1}}{s_m} + \sigma^4 u^2(-1)\frac{s_{l-1}}{s_l}\right] \\
&= \sigma^2(1 + u(-1)^2) - \sigma^2\Big(1 - \frac{1}{s_m}\Big) - \sigma^2\Big(1 - \frac{1}{s_m}\Big),
\end{aligned}$$

and

$$m = \left[{z^-}^{\mathrm{T}}(\sigma^2 A_m A_m^{\mathrm{T}})^{-1} v^- + {z^+}^{\mathrm{T}}(\sigma^2 A_l A_l^{\mathrm{T}})^{-1} v^+\right].$$

This proves the statement of the lemma. ■

The case of Lemma 4.2 where either m or l equals zero can be proven analoguously and is indeed contained in the statement of Lemma 4.2 in the sense that $s_0 = 1$. However, to avoid confusion let us reformulate the case $m = 0$.

Lemma 4.3. *Let $d = 1$, $l \geq 1$, $\Theta = \{-1, 0\}$, $u(0) = 1$ and ρ be the Gaussian density with mean zero and variance σ^2. Let further $v \in \mathbb{R}^l$. Then the distribution of $V_\omega(0)$ conditioned on $(V_\omega(k))_{k=1}^l = v$ is Gaussian with variance*

$$\gamma = \sigma^2 \Big(u(-1)^2 + \frac{1}{s_l} \Big) \quad \text{and mean} \quad m = u(-1) \sum_{i=1}^{l} (A_l A_l^{\mathrm{T}})^{-1}(1,i)\, v_i .$$

Remark 4.4. We want to discuss the validity of the regularity assumption from [ASFH01] in the case $d = 1$, $\Theta = \{-1, 0\}$, $u(0) = 1$ and ρ the Gaussian density function with mean zero and variance σ^2. Notice that the Gaussian distribution is τ-Hölder continuous with a constant C independent on the mean but depending on the variance, and the property that $C \to \infty$ if the variance goes to zero.

Let $l, m \geq 1$. If $|u(-1)| \neq 1$, Lemma 4.2 and Lemma 4.3 gives that the distribution of $V_\omega(0)$ conditioned on fixed potential values $V_\omega(k)$, $k \in \{-m, \dots, l\} \setminus \{0\}$, is again Gaussian with variance bounded from below by $\sigma^2 |u^2(-1) - 1|$. As a consequence, the random field $V_\omega(k)$, $k \in \{-m+1, \dots, n-1\}$ is uniformly τ-Hölder continuous and the constant C from Definition 2.1 may be chosen independently from $m, l \in \mathbb{N}$. Hence the method from [ASFH01] applies and gives localization.

If $|u(-1)| = 1$ the situation is somehow different. In this case Lemma 4.2 and Lemma 4.3 give that the random field $V_\omega(k)$, $k \in \Lambda_L = \{-L, \dots, L\}$, $L \in \mathbb{N}$, satisfies

$$\sup_{x \in \Lambda_L} \sup_{v \in \mathbb{R}^{2L}} \mathbb{P}(V_\omega(x) \in [a,b] \mid V_\omega(k) = v_k, k \in \Lambda_L \setminus \{x\}) \leq C_L (b-a)^\tau$$

but the constant C_L cannot be chosen uniformly in $L \in \mathbb{N}$. In particular, $C_L \to \infty$ if $L \to \infty$. As a consequence, the method of [ASFH01] will give a bound on the expectation on $|G_{\Lambda_L}(z; i, j)|^s$ (,the Green function of the finite volume restriction H_{Λ_L} of the operator H_ω,) which depends on the volume of Λ_L, and hence does not immediately yield localization. If one considers finite volume restriction H_{Λ_L}, an analogue condition to Definition 2.1 which is sufficient for localization would be the following: There is a $\tau \in (0, 1]$ and a constant C such that

$$\sup_{L \in \mathbb{N}} \sup_{x \in \Lambda_L} \sup_{v \in \mathbb{R}^{2L}} \mathbb{P}(V_\omega(x) \in [a,b] \mid V_\omega(k) = v_k, k \in \Lambda \setminus \{x\}) \leq C (b-a)^\tau$$

for all $[a, b] \subset \mathbb{R}$. This condition is obviously not satisfied if $|u(-1)| = 1$ by Lemma 4.2 and Lemma 4.3.

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